BirZeit University
Faculty of Science-Department of Physics
Quantum Mechanics Phys635
Spring 2017
First Exam, Apr. 20th 2017

1. If $|i>$ and $|j>$ are eigenkets of Hermitian operator A. Under what conditions is $(|i> + |j>)$ an eigenket of A

2. Let $|\phi_n>$ be the eigenstate of the Hamiltonian of an arbitrary physical system. Assume the the states $|\phi_n>$ form a discrete orthonormal basis. The operator U(m,n) is defined by:

$$U(m,n) = |\phi_m><\phi_n|$$

   (a) Calculate $U^{\dagger}(m,n)$
   (b) Calculate the commutator [H, U(m,n)]
   (c) Show that:

$$U(m,n)U^{\dagger}(p,q) = \delta_{nq}U(m,p)$$

   (d) Calculate Tr(U(,m,n))
   (e) If A is an operator and $A_{mn} =< \phi_m|A|\phi_n >$ show that

$$A = \sum_{n,m} A_{mn}U(m,n)$$

   (f) Show that $A_{pq} = Tr(AU^{\dagger}(p,q))$

3. Using

$$< x^{'}|p^{'} >= \frac{1}{2\pi\hbar}e^{ip^{'}x^{'}/\hbar}$$

   Prove that:

$$< p^{'}|x|\alpha >= i\hbar\frac{\partial}{\partial p^{'}} < p^{'}|\alpha >$$

4. Two observables A_1 and A_2, which do not involve time explicitly, are known not to commute:

$$[A_1, A_2] \neq 0$$

   yet we also know that A_1 and A_2 both commute with the Hamiltonian:

$$[A_1, H] = [A_2, H] = 0$$

   Prove that the energy eigenstates are, in general, degenerate.

5. A one-dimensional harmonic oscillator is composed of a particle of mass m, charge q and potential energy $V(x) = \frac{1}{2}m\omega^2 X^2$. If the particle is placed in a time-dependent electric field $\varepsilon(t)$ that is parallel to the x-axis, thus creating an additional term in the potential energy $W(t) = -q\varepsilon(t)X$

   (a) Write the Hamiltonian H(t) of the particle in term of a and $a^\dagger$.

   (b) Calculate $[H, a]$ and $[H, a^\dagger]$

   (c) Let $\alpha(t) =< \psi(t)|a|\psi(t) >$ show that:

$$\frac{d}{dt}\alpha(t) = -i\omega\alpha(t) + i\frac{q}{\sqrt{2m\hbar\omega}}\varepsilon(t)$$

   (d) Find $< X > (t)$ and $< P > (t)$

6. Consider an electron of a linear triatomic molecule formed by three equidistant atoms. We use $|\phi_A >$, $|\phi_B >$, $|\phi_C >$ to denote three orthonormal states of this electron, corresponding respectively to three wave functions localized about the nuclei of atoms A, B, C. When we neglect the possibility of the electron jumping from one molecules to another, its energy is described by the Hamiltonian H_0 whose eigenstates are the three states $|\phi_A >$, $|\phi_B >$, $|\phi_C >$. The coupling between the states $|\phi_A >$, $|\phi_B >$, $|\phi_C >$ is described by an additional Hamiltonian W defined by:

$$W|\phi_A >= -a|\phi_B >$$
$$W|\phi_B >= -a|\phi_A > -a|\phi_C >$$
$$W|\phi_C >= -a|\phi_B >$$

   (a) Calculate the energies and stationary states of the Hamiltonian $H = H_0 + W$.

   (b) The electron at time t = 0 is in the state $|\phi_A >$. Discuss qualitatively the localization of the electron at subsequent times. Are there any values of t for which it is perfectly localized about atom A, B or C?

   (c) Let D be the observable whose eigenstates are $|\phi_A >$, $|\phi_B >$, $|\phi_C >$ with respective eigenvalues d, 0, d. D is measured at time t; what values can be found, and with what probabilities ?

   (d) When the initial state of the electron is arbitrary, what are the frequencies that can appear in the evolution of $< D >$? Give a physical interpretation of D. What are the frequencies of the electromagnetic waves that can be absorbed or emitted by the molecule?